Mathematics and the Framework of Quantum Mechanics

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Opening quote

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Functional Analysis Cheat Sheet

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- An operator is unitary if $U^* U = UU^* = I$ ($\implies \langle Ux, Uy \rangle = \langle x, y \rangle$)

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- $\mathcal{H}_1 \otimes \mathcal{H}_2$ is not a Hilbert space if $\mathbb{F} = \mathbb{H}$

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- A divorce of mathematics and physics?

Quick question

Can you hear the shape of a drum?

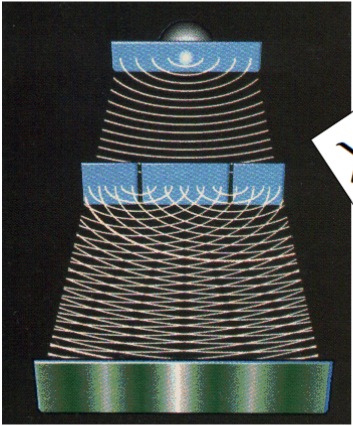
$(\implies \Delta u = \lambda u)$

(Eigen= German for "inherent/characteristic")

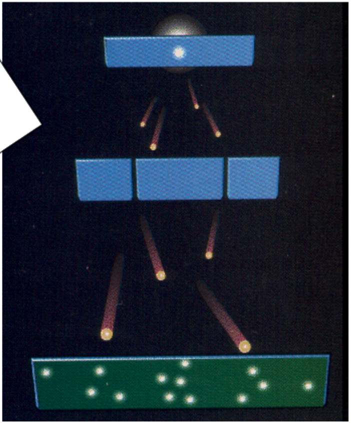
Physical Motivations

$$E = nhf \text{ (not intensity=} W/m^2\text{)}$$

Physical motivations



$$\lambda = \frac{h}{p}$$



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- Axiom 3: Observables = self-adjoint operators

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- $E = i\frac{h}{2\pi}\frac{\partial}{\partial t}$ vs $E = -\frac{\partial}{\partial t} \implies \mathbb{C}$ (yay?)

Axioms of Quantum Mechanics

- Axiom 4: Expectation value $= \langle x, Ax \rangle (= \int \overline{x} A x d\mu$, by the way)
- Composite systems$= \mathcal{H}_1 \otimes \mathcal{H}_2$
- Focus: $\frac{(0,1)+(1,0)}{\sqrt{2}}$ (trouble?)
- Note: if $\mathcal{H} = L^2 [0, 1]$, $\mathbf{x} =$ wave function
- Schrodinger's equation: $H\mathbf{x} = E\mathbf{x}$ (Axiom 5)
- $E = i \frac{h}{2\pi} \frac{\partial}{\partial t}$ vs $E = -\frac{\partial}{\partial t} \Longrightarrow \mathbb{C}$ (yay?)
- 5', $x(t + t_0) = U(t)\, x(t_0)$

Where do these come from?

- $P_{ij} = P\left(\psi_i \longrightarrow \phi_j\right)$ with the following assumptions:

1. $\sum P_{ij} = 1$
2. $P\left(\psi_i \longrightarrow \psi_j\right) = \delta_{ij}$
3. $P\left(\psi_i \longrightarrow \phi_j\right) = P\left(\phi_j \longrightarrow \psi_i\right)$

Any proposed dynamical law, apart from being able to determine probabilities $P\left(\psi_i \longrightarrow \phi_i\right)$ and $P\left(\phi_i \longrightarrow \chi_i\right)$ must also apply to a third, compatible quantity with eigenvectors χ_1, χ_2

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- Why separable? (uncountable eigenvectors)
- Why associative law?

Alternative Formulations of Quantum Mechanics

- Path Integral Formulation
- Phase space formulation of Quantum Mechanics and Geometric Quantization
- Signed Particle Formulation
- Quantum Field Theory in Curved Spacetime
- Axiomatic, Algebraic and Constructive Quantum Field Theory
- $C*$-algebraic formalism
- Generalized Statistical Model of Quantum Mechanics